Parallel Capacitors Having Loss.

We've all learned that when you have two capacitors in parallel, the total capacitance is the sum of the two capacitances. Not necessarily so, though the approximation generally serves us well. For exact results, capacitor losses must also be taken into account.

In the following discussion we'll talk about both the parallel and series capacitance models, and about parallel and series combinations of actual capacitors, which could be specified using either model, so follow the text closely.

First, let's consider two ideal capacitors with a resistance in series with each one to represent losses. Both the values and losses will be assumed to be at least slightly different to avoid divide-by-zero problems in the formulas to come. We'll put the two series model capacitors in parallel.

Both the equivalent series capacitance of the pair (Cs_{tot}) and the equivalent parallel capacitance (Cp_{tot}) will be *less* than the sum of the two capacitances.

Obviously the situation is reversed if starting with the parallel model. The equivalent parallel capacitance of the pair is exactly the sum of the two capacitances, but the equivalent series capacitance will be greater than the sum because $Cs = Cp(1 + D^2)$. Here $D = 1/\omega RC$ where R and C parallel elements.

The models cover all sources of loss. Typically, the major source of loss is dielectric loss, a characteristic of the particular dielectric material chosen, with a very small component being leakage and the internal resistance of leads and connections. However, whichever model one might use, the values given for C and D define an equivalent series resistance (ESR) from the relationship that $R = D/\omega C$. This simple series R-C model is valid for only one frequency; if the frequency is changed, the R and C values must be recalculated. Note that the dissipation factor (D) is a ratio, a measure of efficiency that does not change, regardless of which model is chosen.

Consider Equation 1 where both C and R are ideal elements in series. The simplest calculation is for the total parallel capacitance. Letting the dissipation factor (D) equal ωRC, we have

$$Cp_{tot} = C_1/(1 + D_1^2) + C_2/(1 + D_2^2) \qquad [1]$$

If either D is greater than zero the denominator becomes greater than one and Cp_{tot} must therefore be less than the sum of $C_1 + C_2$.

The equivalent series capacitance of the pair is a little trickier. First, if D_1 and D_2 are equal, then indeed $Cs_{tot} = C_1 + C_2$. However, if D_1 and D_2 are not equal, Cs_{tot} must again be less than $C_1 + C_2$ as shown in equation 2:

$$Cs_{tot} = (C_1 + C_2)[1 + (C_1D_2 + C_2D_1)^2 / (C_1 + C_1)^2] / [1 - D_1D_2 + (D_1 + D_2)(C_1D_2 + C_2D_1)/(C_1 + C_2)] \qquad [2]$$

$$D_{tot} = [C_1D_1 + C_2D_2 + D_1D_2(C_1D_2 + C_2D_1)] / (C_1 + C_2 + C_1D_2^2 + C_2D_1^2) \qquad [3]$$

The equations are much simplified if one of the dissipation factors is zero.

It isn't difficult to see that Cs_{tot} is often less than $C_1 + C_2$, but it's quite unexpected that Cs_{tot} can actually be smaller than either C value making up the parallel combination. In other words, if the condition of equation 4 is met, the addition of a smaller low-loss capacitor can actually reduce the value of a larger high loss capacitor when the series model is used [1, 2]:

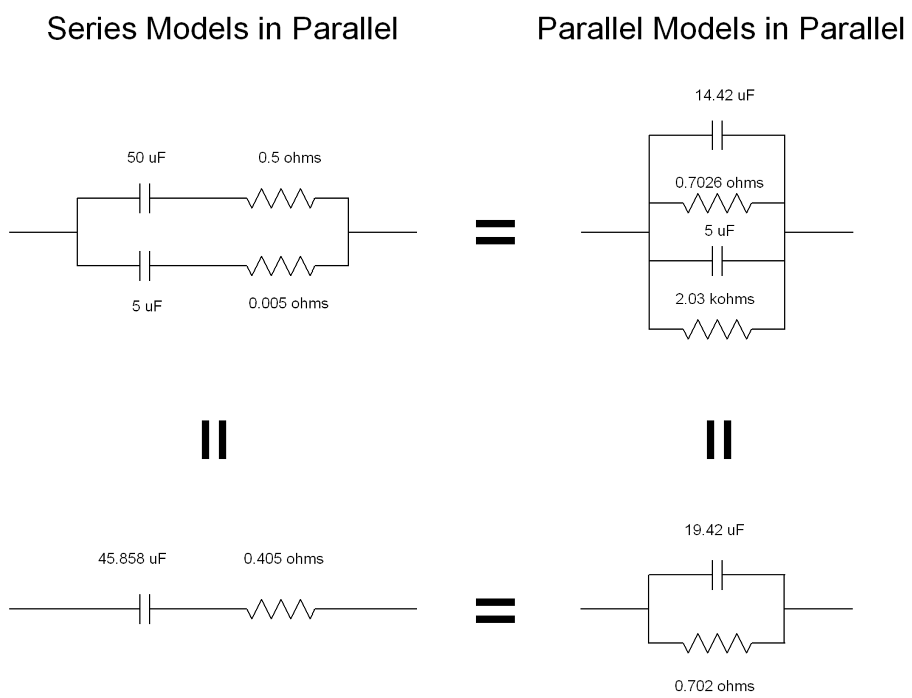
$$Cs < C_1 \text{ if } D_1 > [C_1D_2 + \mathrm{Sqrt}(C_1^2D_2^2 + (C_1+C_2)(C_1 - C_2))] / (C_1 - C_2) \qquad [4]$$

In this formula C1 is assumed to be the larger capacitance. The formula is invalid if $C_1 = C_2$.

The above equations yield some useful rules:

1) If either or both capacitors have loss, $Cs_{tot} < C_1 + C_2$, except when $D_1 = D_2$.
2) If $D_1 = D_2$ then $Cs_{tot} = C_1 + C_2$.
3) If $D_1 \neq D_2$ and $C_1 \neq C_2$ then $Cs_{tot} < C_1$ if $D_1 > [C_1D_2+\mathrm{Sqrt}(C_1^2\ D_2^2+(C_1+C_2)(C_1-C_2))]/(C_1-C_2)$
4) If $C_1 = C_2$, Cs_{tot} can't be less than C_1 (D_1 or D_2 would have to be infinite)

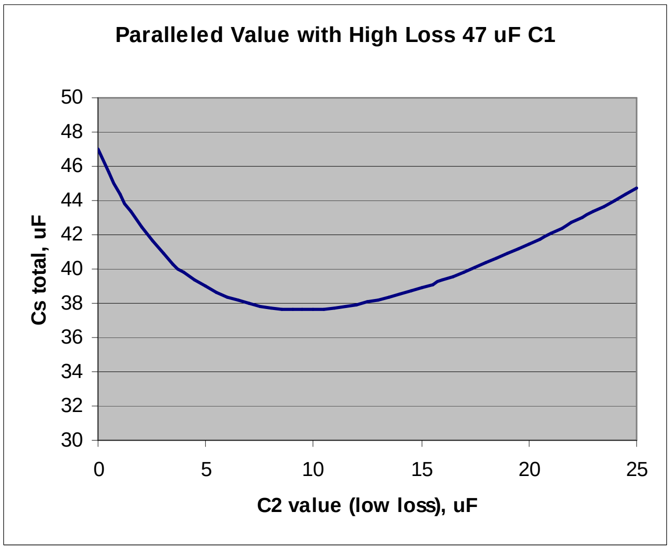
Model Review



Physically, all the models represent a hypothetical 50 uF capacitor having a dissipation factor of 1.5708, connected in parallel with a 5 uF capacitor having a dissipation factor of 0.00157, both measured using the series model, at 10 kHz. The combined dissipation factor is 1.167

The Interesting Region of Rule 3

This chart shows the region where a high loss part is combined with a low loss part, in this case the calculated Cs_{tot} for a fixed 47 uF capacitor having a dissipation factor of 2, paralleled with a low loss capacitor ranging from 0 to 25 uF.



Measurements

Various actual parts were chosen and measured under conditions designed to illustrate some of the results described above. The losses for the first pair were created by adding resistance, due to the difficulty of finding parts with matched dissipation factor, to illustrate that the values total as expected. The remaining parts had the actual losses shown at the frequencies tested. All measurements agreed closely with the calculated results.

Frequency	C_1, uF	D_1	C_2, uF	D_2	Cs_{tot} meas.	Cs_{tot} calc.	D_{tot} meas.	D_{tot} calc.	D Eqn [4] Rule 2	Result
1 kHz	4.973	1.200	2.975	1.200	7.949	7.948	1.200	1.200	6.578	$Cs_{tot} = C_1 + C_2$
1 kHz	42.4	0.227	5.12	0.00053	47.23	47.29	0.209	0.2015	1.130	$Cs_{tot} < C_1 + C_2$
5 kHz	1190	1.949	145.04	0.1318	1036	1036	1.285	1.286	1.290	$Cs_{tot} < C_1$
15 kHz	34.76	2.18	5.13	0.0074	26.07	26.79	1.194	1.183	1.169	$Cs_{tot} < C_1$

N Capacitors

The formulas presented handle combinations of two capacitors. In the event that multiple capacitors are paralleled, simply convert each Cs to Cp and place the models in parallel. Sum the capacitances and parallel the resistances. Convert the resulting Cp back to Cs.

$$Cp=Cs(1/(1+D^2)) \qquad Cs=Cp(1+D^2) \qquad Rp=1/(\omega CpD) \qquad D=1/(\omega CpRp) \qquad [5]$$

Discussion

One should be mindful of unexpected results if the series model is chosen and parallel combinations with high losses are involved.

Older mechanical bridges will be limited to one or the other model, depending on losses. Typically there will be a low D and high D range, with a bit of overlap, associated with Cs and Cp, respectively, thus insuring that one uses the parallel model for high loss capacitors. Digital impedance meters usually measure both Cs and Cp for high and low loss capacitors, leaving the choice entirely up to the user.

High losses would be common with aluminum electrolytic capacitors operating above a few kHz. Another possible situation is when small resonant piezoelectric actuators and energy harvesters are operated at the end of a low loss cable. At resonance the losses in the piezoelectric device may be very high compared to the cable and the measurement issues described here can easily arise when one tries to reconcile the total capacitance with the piezoelectric device and cable measured separately.

Conclusion

The common formulas for parallel and series capacitor combinations do not take losses into account. When losses are low this is of little or no consequence. When losses are higher, the formulas presented here will give exact results for parallel combinations. Again, it should be noted that Cp is a better choice for high loss calculations as it avoids the non-intuitive results described here for the Cs model. For completeness, similar formulas would also apply for inductors, but it would be extremely rare for anyone to parallel inductors.

Acknowledgements

To Eric Hauptman, who double checked the initial results, and to Henry P. Hall, formerly of General Radio Corp. (later GenRad) who derived the formulas given here, and provided significant guidance and encouragement.

C. Hoffman,
8/18/2015 (fixed delimiters in eqn 4)

[1] This unexpected result was noticed while investigating various high-loss/low-loss capacitor bypass schemes commonly used by audiophiles in amplifiers and crossovers. It’s hard to believe that the phenomenon hasn’t been previously noted. If you have knowledge of any other reference, please let us know.

[2] Henry hadn’t noticed this effect, and if it’s possible that it hasn’t been described before, suggested we call it the “Hoffman Effect” (there already is a Hall Effect).